

Lagrangian Temperature, Velocity, and Local Heat Flux Measurement in Rayleigh-Bénard Convection

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We have developed a small, neutrally buoyant, wireless temperature sensor. Using a camera for optical tracking, we obtain simultaneous measurements of position and temperature of the sensor as it is carried along by the flow in Rayleigh-Bénard convection, at $Ra \sim 10^{10}$. We report on statistics of temperature, velocity, and heat transport in turbulent thermal convection. The motion of the sensor particle exhibits dynamics close to that of Lagrangian tracers in hydrodynamic turbulence. We also quantify heat transport in plumes, revealing self-similarity and extreme variations from plume to plume.

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Understanding fluid motion and transport of heat due to thermal convection is crucial for progress in diverse challenging and important problems such as climate change, processes in planetary and stellar cores, and efficient temperature control in buildings. Numerous laboratory studies employing Rayleigh-Bénard experiments have uncovered laws which relate global heat flux and flow velocities to fluid properties, flow boundary geometry, and driving parameters [1]. More recently, several studies [2,3] have focused on coherent flow structures, called plumes, which are ejected from the thermal boundary layers carrying heat into the convective flow. We present measurements from a novel temperature sensor which is carried along with the convective flow, i.e., Lagrangian measurements. In this way we obtain statistics of simultaneous velocity, temperature, and heat transport dynamics throughout the lifetimes of many plumes.

Lagrangian measurements are particularly well suited for studying flows where coherent structures [4] and mixing [5] are important, e.g., plumes mixing temperature in convection. Recent measurements and numerical simulations of high Reynolds number hydrodynamic turbulence have shown the importance of the long-time effects in the Lagrangian dynamics [6]. In the context of convection, Lagrangian measurements have long been performed using meter-sized atmospheric balloons [7] or ocean floats [8]. Here we use a similar strategy, aided by advances in miniature sensors and communication devices [9], to probe temperature and flow properties at centimeter scales in Rayleigh-Bénard convection.

Our experimental setup is a rectangular vessel with height $H = 40$ cm and section 40 cm \times 10 cm filled with water (for more detail, see [10]). The walls are made of polymethylmetacrylate (PMMA), the top boundary is a copper plate chilled by a controlled water bath, and the bottom boundary is an electrically heated copper plate. The heater power is maintained at 230 W and the top plate is held at $T_{\text{up}} = 19$ °C, resulting in a temperature difference $\Delta T = 20.3$ °C between the top and bottom plates. The resulting Rayleigh number is $Ra = g\beta\Delta TH^3/\nu\kappa = 3 \times$

10^{10} , where g is acceleration due to gravity, $\beta = 2.95 \times 10^{-4}$ K $^{-1}$ is the thermal expansion coefficient of water and $\nu = 8.17 \times 10^{-7}$ m 2 s $^{-1}$, $\kappa = 1.48 \times 10^{-7}$ m 2 s $^{-1}$ its viscosity and thermal diffusivity (values are given for the mean temperature of the flow 29.1 °C). The Nusselt number, defined as the total heat flux normalized by $\lambda\Delta T/H$, $Nu = 167.9 \pm 0.2 - \lambda = 0.6142$ W m $^{-1}$ K $^{-1}$ is the thermal conductivity of water. Under these conditions, the convective regime is fully turbulent [10–12] and the mean flow is a steady, system-sized, convection roll with a rotation period of about 100 s. For the duration of the experimental runs, we have not observed spontaneous reversals of the large scale convection roll.

The mobile sensor consists of a $D = 21$ mm diameter capsule containing temperature instrumentation, an rf emitter, and a battery. It is described in detail in [9] and we recount its basic features here. The capsule and fluid density are carefully matched within 0.05% so that it reliably follows the flow. Four thermistors (0.8 mm, 230 k Ω , response time 0.06 s in water) are mounted in the capsule wall protruding 0.5 mm into the surrounding flow. A resistance controlled oscillator is used to create a square wave whose frequency depends on the temperature of the thermistors. This square wave is used directly to modulate the amplitude of the radio wave generated by the rf emitter. The temperature signal is recovered on the fly by a stationary receiver and a Labview program. The dynamic range of temperature detection is 80 dB, with a resolution of 4 mK and 50 ms. In addition, the capsule trajectory is recorded with a digital video camera, providing synchronous measurements of the position and temperature of the sensor as it is carried about by the fluid. With maximum flow velocities in the range 1–2 cm/s and a particle size of 21 mm, we are oversampling the dynamics by a factor of order 10. However, the characteristic thickness of the thermal boundary layer is $\ell_T \sim \frac{1}{2}H\nu^{-1} \sim 1.2$ mm and that of the hydrodynamic boundary layer may be estimated [1] as $\ell_U \sim \ell_T(\nu/\kappa)^{+1/3} \sim 2$ mm. Thus, the sensor is too large to penetrate the boundary layers.

Before presenting more detailed statistics, we provide an overview of the raw data collected by the mobile sensor. Predominantly, the sensor moves in looping trajectories the size of the convection cell with a period of about 100 s. The collective result of many fluid parcels (plumes) with such trajectories compose the large scale convection roll—one may also detect in Fig. 1(a) the presence of secondary rolls in the lower-left and upper-right corners. Although the main motion of the sensor is rather periodic, we point out that its temperature fluctuates widely and irregularly [see Figs. 1(b)–1(d)]. Figure 1(e) shows the cross correlation between temperature and normalized velocity. We find a maximum value of 0.6 at zero time lag and confirmation of the 100 s period of the large scale roll. This correlation is about twice the value reported in [13] for local Eulerian measurements. We attribute these dynamics to the entrainment of the sensor by thermal plumes and associate the fluctuations in Figs. 1(b)–1(d) with variation between plumes. In addition to qualitative verification of this idea using Schlieren visualizations, it is consistent with previous studies which find plumes predominately near the sidewalls like the trajectory of our sensor [13,14]. We explore these dynamics quantitatively in the results to follow.

From the data in Fig. 1, one may estimate characteristic dimensionless numbers for the flow. From integral quantities one computes the integral Reynolds number $Re = UH/\nu \sim 4000$. The associated turbulent—Taylor based—Reynolds number is of the order of $R_\lambda \propto \sqrt{Re} \sim 63$. We note that this value is very close to a local Reynolds number, defined from the actual motion of the Lagrangian sensor $Re' = u_{rms} \ell_{rms}/\nu \sim 60$, where ℓ_{rms} is the fluctuation of position along a “mean” trajectory, u_{rms} is the usual standard deviation of the velocity. In this regime ($R_\lambda \sim 60$), one expects turbulence with significant intermittency in the velocity gradients [15].

Accordingly, we turn now to more detailed statistics of our measurements including power spectra, increment probability distribution functions (PDF), and structure

function scaling for velocity and temperature. The power spectra (in time) are shown in Fig. 2(a). The velocity spectrum is close to the behavior expected for a Lagrangian tracer in a turbulent flow— f^{-2} —although scaling range is limited at such low R_λ . The temperature spectra is observed to roughly mimic that of velocity with a steeper power law slope compared to measurements with a stationary temperature probe. For such Eulerian measurements, experiments [16] measure slopes between -1.35 and -1.4 ($\equiv -7/5$) while numerical studies [17] report $-7/5$ in the bulk and $-5/3$ near the sidewalls. Velocity increments [Fig. 2(b)] indicate that the sensor is subjected to rather intermittent acceleration with strongly non-Gaussian statistics—flatness is -21 . Like the power spectrum, the shape of PDFs for different velocity increments is similar to those of a fluid particle in a turbulent flow—Fig. 2(b). Finally, we have computed the evolution of the structure functions for the vertical velocity $S_{v_y}^p(\tau) = \langle |v_y(t+\tau) - v_y(t)|^p \rangle_t$; we observe an extended region of relative scaling $S_{v_y}^p(\tau) \propto [S_{v_y}^2(\tau)]^{\xi_p}$ (for $0.3 \leq \tau \leq 10$ s) with exponents $\xi_1 = 0.56$, $\xi_2 = 1.00$, $\xi_3 = 1.30$, $\xi_4 = 1.50$, $\xi_5 = 1.65$, $\xi_6 = 1.7$. These values (determined with a 10% precision) are again in good agreement with experimental measurements for Lagrangian tracers in turbulence [6]. We also show in Fig. 2(c) the PDF of temperature increments measured by the mobile sensor. Similar to Eulerian measurements [13], their statistics are non-Gaussian, with wider tails at small scale. In contrast to the velocity behavior (and to some Eulerian studies [17]), our Lagrangian temperature increments do not reveal a range of self-similar scaling.

As proposed in several previous studies [14,18,19], the dynamics of heat transport may be best analyzed in terms of a local heat flux $v_y \theta'(t)$, where $\theta'(t) = \theta(t) - \bar{\theta}$ is the particle temperature variation from its time averaged temperature $\bar{\theta}$, and $v_y(t)$ its vertical velocity. Our mobile sensor allows for a Lagrangian measurement of this quantity. In particular, we define the normalized Lagrangian horizontal and vertical heat transports as

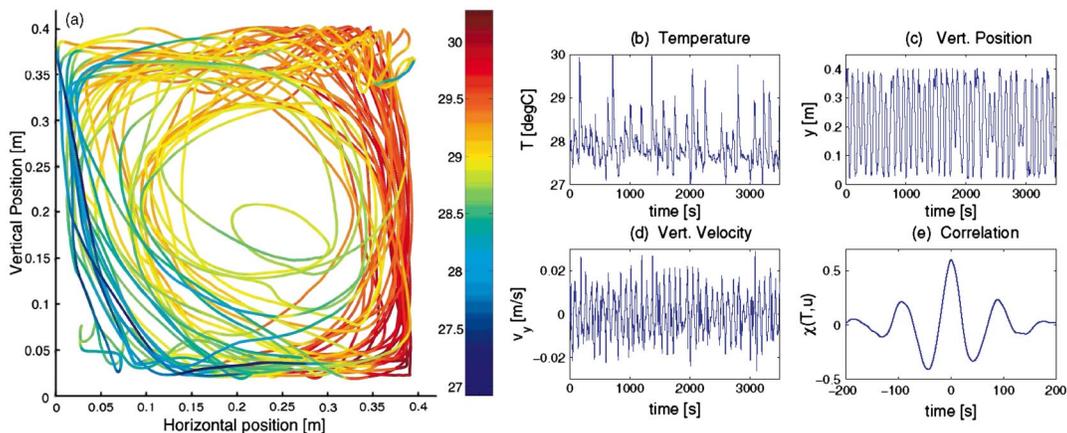


FIG. 1 (color online). Temperature and velocity measurements of the mobile sensor. (a) Trajectory with temperature color coding, (b) temperature, (c) vertical position, (d) vertical velocity, (e) cross correlation of temperature and velocity.

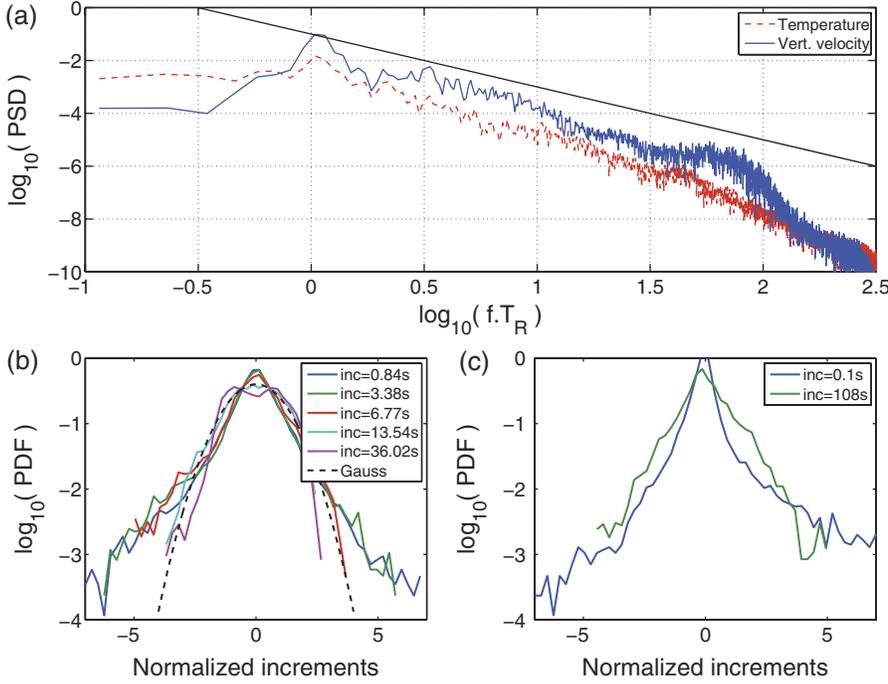


FIG. 2 (color online). Temperature and velocity statistics. (a) Time spectra of temperature and vertical velocity of the sensor particle; the frequency is nondimensionalized using the period $T_R = 100$ s of the large scale roll motion (curves have been shifted vertically for clarity). The solid straight line corresponds to an ideal f^{-2} scaling. (b) PDFs of vertical velocity increments (time lags are given in the inset), (c) PDFs of temperature increments for time lags equal to 0.1 and 102.4 s.

$$\text{Nu}_{x,y}^L(t) = 1 + \frac{H}{\kappa \Delta T} v_{x,y}(t) \theta'(t). \quad (1)$$

The time series of $\text{Nu}_y^L(t)$ are shown in Fig. 3(a). As expected, it is most often positive since convective motions are associated with either hot fluid rising or cold fluid sinking, in each case $(v_y, \theta') > 0$. The much less probable events with $(v_y, \theta') < 0$ correspond, for instance, to the rise of the particle when it is colder than its environment due to turbulent swirls in the flow. We find that the time averaged Lagrangian heat transport $\overline{\text{Nu}_y^L} = 328$ is larger than the global value ($\text{Nu} = 168$ in our case). This indicates that our sensor preferentially samples the regions of the flow which carry higher than average heat, i.e., plumes. In contrast, the traditional Nusselt number indiscriminately accounts for all regions in the flow such as the center of the cell where plumes rarely visit. Another prominent feature is the highly non-Gaussian, intermittent fluctuations of Lagrangian heat transport in the horizontal and vertical directions. Their normalized probability density functions are shown in Figs. 3(c) and 3(d). They are found to be in remarkable agreement with previous Eulerian measurements with stationary probes at similar Rayleigh numbers. The statistics of horizontal heat flux variations is in agreement with the stretched exponential distributions proposed in [13] and found to be independent of Ra in the range $[10^9 - 3 \times 10^{10}]$. The vertical heat flux statistics is almost exactly identical to the Eulerian measurement near the sidewall in [13] and interpreted in terms of transport by thermal plumes [14,18].

As for the velocity, we have studied the statistics of the increments (in time) of the vertical heat transport $\text{Nu}_y^L(t)$. Comparison with the velocity shows that the Lagrangian

heat flux is less intermittent; i.e., the PDFs of the local heat flux increments are non-Gaussian but their shapes do not evolve significantly as increments increase towards the large scale period T_R .

A simple interpretation of the above statistical results is that thermal plumes may be defined as Lagrangian heat transport events (once detached from the thermal boundary layer); these events are in some way self-similar as will

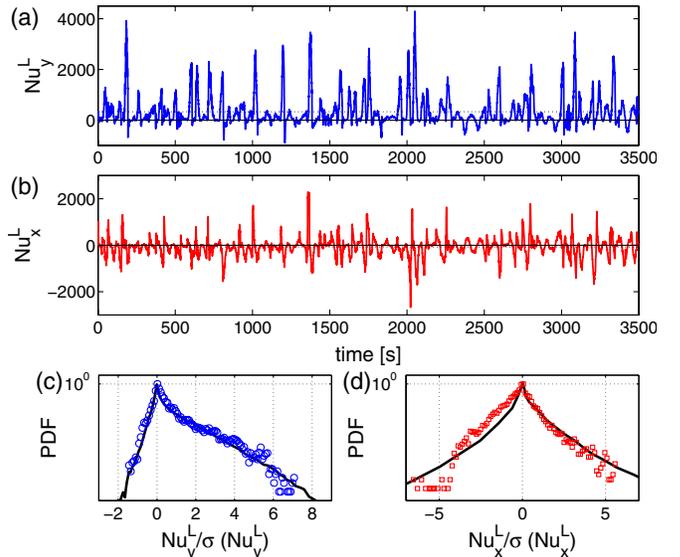


FIG. 3 (color online). Lagrangian heat transport. (a),(b) Time series of the vertical and horizontal heat flux (the dashed line corresponds to the mean values $\overline{\text{Nu}_y^L} = 328$, $\overline{\text{Nu}_x^L} = -78$), (c),(d) corresponding PDFs for the signals normalized by their variance. The thick solid line—from [13]—corresponds to Eulerian measurements made near the sidewall.

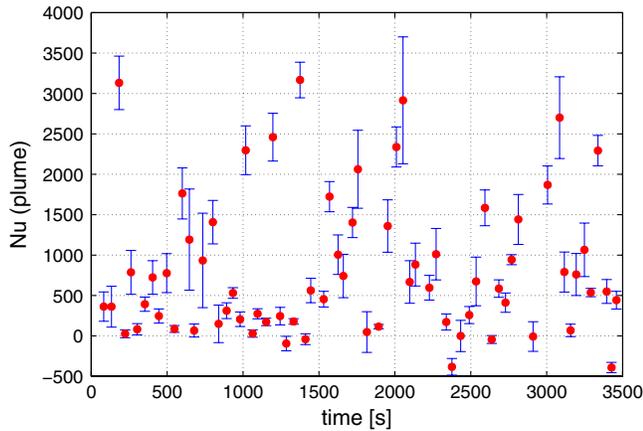


FIG. 4 (color online). Plumes. Mean (dots) and standard deviation (error bars) of Lagrangian heat transport of consecutive plumes.

become clear below. In this light, we have analyzed the portions of the sensor trajectories during which its vertical position lies between $H/4$ and $3H/4$, i.e., the rise (or fall) of a hot (cold) plume. For each of these motions, we compute the mean heat transport, and its fluctuation measured by the standard deviation computed over the portion of the trajectory. The results are reported in Fig. 4. We observe that the heat transport has large variations from one plume to the next. The mean Lagrangian heat transport in these events ($\overline{\text{Nu}}^{\text{pl}} = 817$) which indicates that the times the sensor is not in a plume are relatively unimportant to heat transport. Furthermore, the variation from plume to plume is quite large, $\text{rms}(\text{Nu}^{\text{pl}}) = 866$, of the order of the mean. During the trajectory of a single plume, the standard deviation σ of Nu^{pl} is roughly proportional to the mean: $\sigma \approx \alpha \overline{\text{Nu}}^{\text{pl}}$. We note that the plume heat transport we measure matches within a factor of 4 the prediction of the model proposed in [19], though more experiments are required to test scaling laws for different Rayleigh numbers and fluid properties. Furthermore, we find that the characteristic time for temperature decay during the lifetime of a plume is of the order of its travel time from one plate to the other, which also supports the model in [19].

To summarize, we have reported novel measurements of Lagrangian temperature and heat transport using a wireless, neutrally buoyant temperature sensor. The close agreement between previous analysis of heat transport under the form of coherent motions from Eulerian measurements [13,14,18] and our Lagrangian data recorded by an advected passive tracer give support to the dominant role of plumes in the transport of heat. It also brings a new perspective on the dynamics of these coherent structures in turbulent Rayleigh-Bénard convection. We find that heat transport fluctuates greatly from one plume to another and

that these fluctuations suggest a self-similar character of plumes. A practical implication of our results is that in order to maximize heat transport, one should maximize the production of thermal plumes. This is supported by global Nu versus Ra measurements in experiments [20,21] with rough end plates. Future investigations will be focused on the scaling of plume heat transport for varying Ra and different fluids.

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